On Heterotic Noncompact Nonlinear Sigma Models

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Outline

- N=2 SQCD in 4d & Sigma models in 2d
- GLSM as a tool to find BPS spectrum
- Heterotic deformation and Large-N solution - beyond BPS sector
- Modeling confinement
- Perturbation theory

4d SQCD vs 2d sigma models

4d / 2d duality

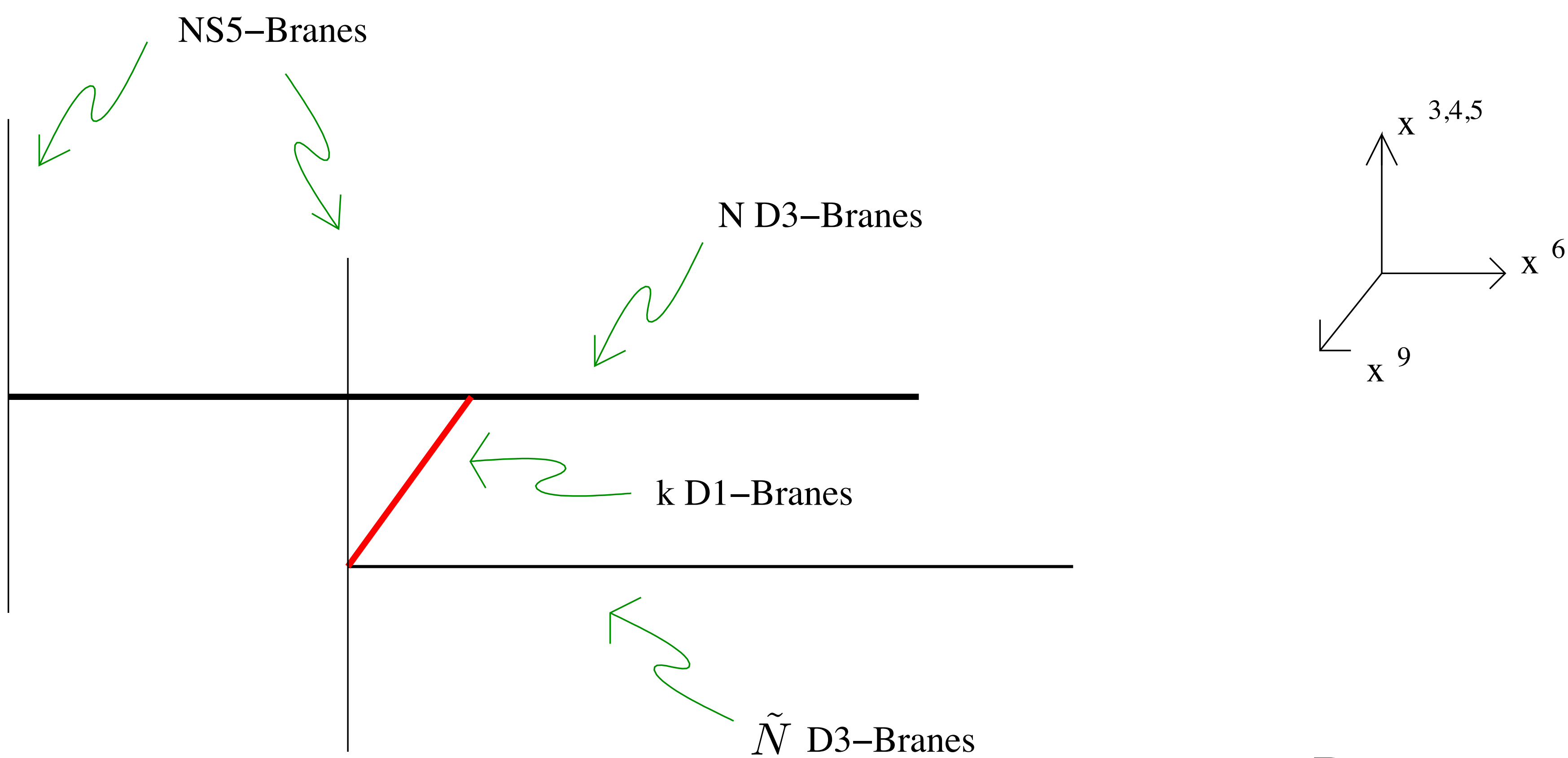
[Dorey Hollowood, Tong]

$\mathcal{N}=2 \quad SU(N)$ SQCD	(2,2) $U(1)$ GLSM e
$N_f = N + \tilde{N}$ fund hypers	N chiral +1 $\quad \tilde{N}$ chiral -1
w/ masses	w/ *twisted* masses
$m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$	$m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$
$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$	$\tau = ir + \frac{\theta}{2\pi}$
on baryonic Higgs branch	vortex moduli space
BPS dyons (Seiberg-Witten)	kinks interpolating between different vacua

BPS spectra (as functions of masses, Lambda) are the same

Brane construction

[Hanany Tong]



D-term

take k=1 to get U(1) theory on D1 (semi-local vortex)

$$|Q_i|^2 - |\tilde{Q}_j|^2 = r$$

FI term -- separation of NS5s in x6

will refer to as HT model

$U(N_c)$ $\mathcal{N}=2$ $d=4$ SQCD w/ N_f quarks

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\beta}\} = 2\delta^{IJ} P_{\alpha\dot\beta} + 2\delta^{IJ} Z_{\alpha\dot\beta}$$

strings

$$\{Q^I_\alpha, Q^J_\beta\} = 2Z^{IJ}_{\alpha\beta}$$

monopoles

$$\mathcal{L} = \mathrm{Im}\left[\tau \int d^4\theta\, \mathrm{Tr}\left(Q^{i\,\dagger} e^V Q_i + \tilde{Q}^{i\,\dagger} e^V \tilde{Q}_i + \Phi^\dagger e^V \Phi\right)\right] + \mathrm{Im}\left[\tau \int d^2\theta \left(\mathrm{Tr} W^{\alpha\,2} + m^i_j \tilde{Q}_i Q^j + Q_i \Phi \tilde{Q}^i\right)\right]$$

bosonic part

FI term

$$S = \int d^4x\, \mathrm{Tr}\left\{\frac{1}{2g^2}F^2_{\mu\nu} + \frac{1}{g^2}|D_\mu\Phi|^2 + |\nabla_\mu Q|^2 + \frac{g^2}{4}(Q\bar{Q} - \xi)^2 + |\Phi Q + QM|^2\right\}$$

BPS conditions

$$B_3 - g^2(Q\bar{Q} - \xi^2) = 0$$

$$\nabla_3 Q = 0$$

String tension

$$T = \xi \int d^2x\, \mathrm{Tr} F_{12} = 2\pi\xi n$$

Vortex moduli space

Nf=Nc color-flavor locked phase
single SUSY vacuum

$$U(N_c) \times SU(N_f) \to SU(N)$$

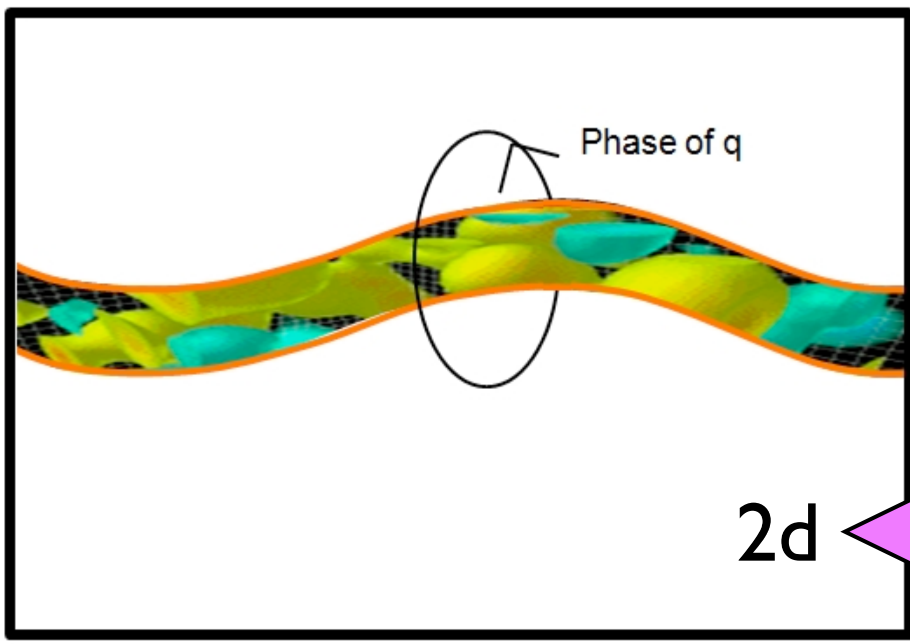
local vortex

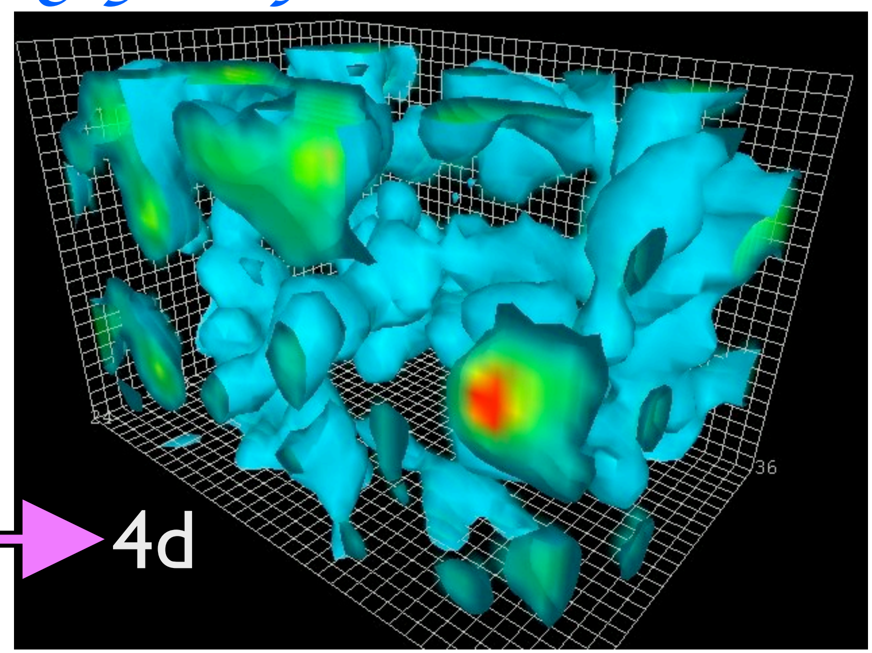
$$\frac{SU(N)}{SU(N-1) \times U(1)} = \mathbb{CP}^{N-1}$$

Nf>Nc semilocal

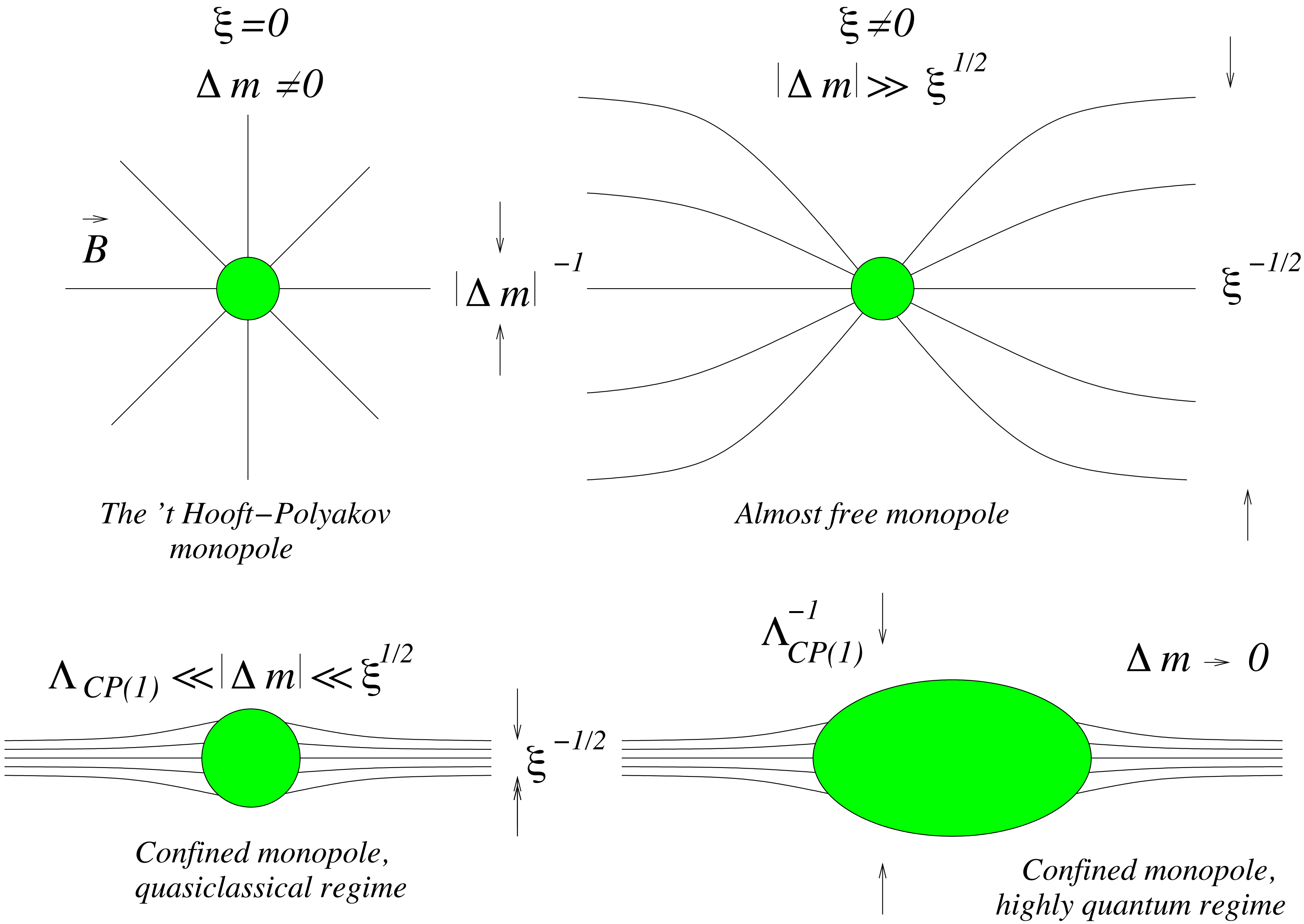
$$\pi_2(\mathcal{M}_{vac}) = \pi_2\left(\frac{SU(N+\tilde{N})}{SU(N) \times SU(\tilde{N}) \times U(1)}\right) = \mathbb{Z}$$

Duality between two strongly coupled theories





Confined monopoles



$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term r

Hanany-Tong model as U(1) GLSM

$$\mathcal{L} = \int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, \mathrm{e}^{\mathcal{V}} \Phi_i + \sum_{i=1}^{\tilde{N}} \widetilde{\Phi}_i^\dagger \, \mathrm{e}^{-\mathcal{V}} \widetilde{\Phi}_i - r\mathcal{V} + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

Limit $e \to \infty$ defines vacuum manifold $\mathcal{O}(-1)^{\otimes \tilde{N}}$ over $\mathbb{CP}^{N-1}$

perturbation theory is subtle

$$\beta_{i\bar{\jmath}} = a^{(1)} R_{i\bar{\jmath}} \log \frac{M}{\mu} + \ldots \qquad R_{i\bar{\jmath}} = \frac{N - \tilde{N}}{r} g_{i\bar{\jmath}} + \mathcal{O}(\frac{1}{r^2})$$

however there are nonperturbative corrections

One loop twisted effective superpotential is exact in (2,2)

$$\widetilde{\mathcal{W}}_{eff} = \frac{N - \tilde{N}}{4\pi} \sigma \left(\log \frac{\sigma}{\Lambda} - 1 \right)$$

$$V = \theta^+ \bar{\theta}^+ (A_0 + A_3) + \theta^- \bar{\theta}^- (A_0 - A_3) - \theta^- \bar{\theta}^+ \sigma - \theta^- \bar{\theta}^+ \bar{\sigma} + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta} \bar{\lambda} + \bar{\theta} \theta \bar{\theta} \theta D$$

gives vacua of the theory and its BPS spectrum !!

Heterotic deformation

(0,2) Theory

[Gorsky Shifman Yung]

In 4d introduce masses $\int d^2\theta\, \mu^2 (\Phi^a)^2$

breaks $\mathcal{N}=2$ to $\mathcal{N}=1$

On the flux tube $(2,2) \longmapsto (0,2)$

obtain heterotic sigma model

[Edalati Tong][Shifman Yung] [Distler Kachru]

$$\mathcal{L} = \int d^4\theta \left(\Phi_i^\dagger e^V \Phi^i - rV - \mathcal{B}V \right)$$

$$\mathbb{CP}^{N-1} \times \mathbb{C}$$

B-right handed superfield

can be treated as model w/ field dependent FI term

$$K = (r + \mathcal{B}) \log(1 + |\phi^i|^2)$$

Geometry becomes non-Kahler

due to generation of $\mathcal{H}$ field (field dependent theta term)

(0,2) GLSM

[PK Monin Vinci]

$$\int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, e^V \Phi_i + \sum_{i=1}^{N_c - N_f} \tilde{\Phi}_i^\dagger \, e^{-V} \tilde{\Phi}_i - (r + \mathcal{B})V + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

$$\Phi^i = n^i + \bar{\theta}\xi^i + \theta\bar{\xi}^i + \bar{\theta}\theta F^i \,, \quad i = 1, \ldots, N_c$$
$$\widetilde{\Phi}^j = \rho^j + \bar{\theta}\eta^j + \theta\bar{\eta}^j + \bar{\theta}\theta \tilde{F}^j \,, \quad j = 1, \ldots, \tilde{N}$$

$$\Sigma = \sigma + i\theta^+ \bar{\lambda}_+ - i\bar{\theta}^- \lambda_- + \theta^+ \bar{\theta}^- (D - iF_{01})$$

$$\mathcal{B} = \omega(\bar{\theta}\zeta_R + \bar{\theta}\theta \bar{\mathcal{F}}\mathcal{F})$$

deformation adds

$$\mathcal{L}^{het} = \mathcal{L} + \bar{\zeta}_R \partial_L \zeta_R - |\omega|^2 |\sigma|^2 - [i\omega\lambda_L\zeta_R + \mathrm{H.c.}]$$

Not enough SUSY to fix superpotential

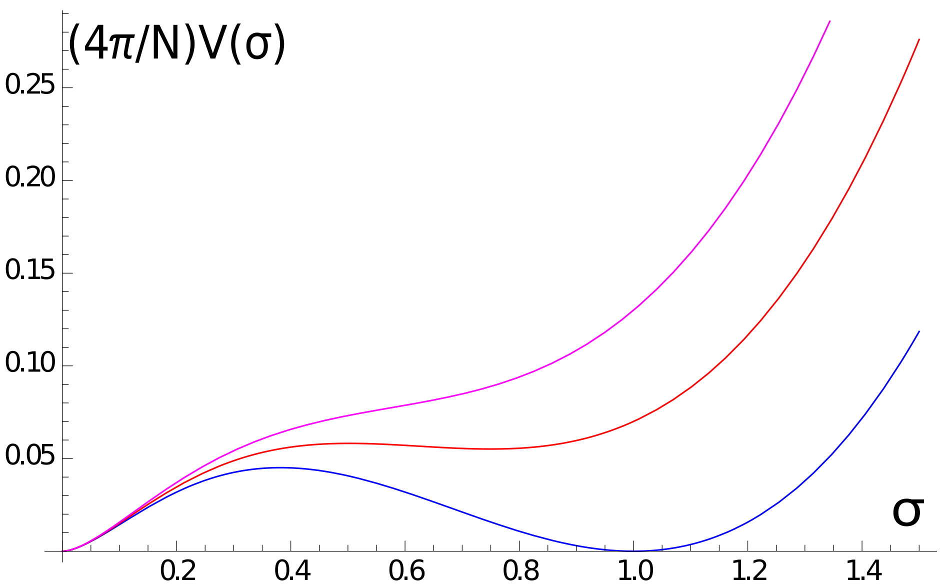
Have to dwell on large-N approach

Large-N solution of (0,2)

$$V_{1-loop} = \frac{1}{4\pi}\sum_{i=1}^{N-1}\left(-\left(D+|\sigma-m_i|^2\right)\log\frac{|\sigma-m_i|^2+D}{\Lambda^2}+|\sigma-m_i|^2\log\frac{|\sigma-m_i|^2}{\Lambda^2}\right)$$
$$-\frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1}\left(-\left(D-|\sigma-\mu_j|^2\right)\log\frac{|\sigma-\mu_j|^2-D}{\Lambda^2}-|\sigma-\mu_j|^2\log\frac{|\sigma-\mu_j|^2}{\Lambda^2}\right)$$
$$+\frac{N-\tilde{N}}{4\pi}D\,.$$

$$V_{eff} = V_{1-loop} + \left(|\sigma-m_0|^2+D\right)|n_0|^2 + \left(|\sigma-\mu_0|^2-D\right)|\rho_0|^2 + \frac{uN}{4\pi}|\sigma|^2$$

for zero masses



Symmetric masses

$$m_k = m\,e^{2\pi i\frac{k}{N}}\,,\quad k=0,\ldots,N-1\,,$$
$$\mu_l = \mu\,e^{2\pi i\frac{l}{\tilde{N}}}\,,\quad l=0,\ldots,\tilde{N}-1\,.$$

Vacuum equations

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0\,,$$

$$\frac{1}{4\pi}\sum_{i=1}^{N-1} \log \frac{|\sigma - m_i|^2 + D}{\Lambda^2} - \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} \log \frac{|\sigma - \mu_j|^2 - D}{\Lambda^2} = |n_0|^2 - |\rho_0|^2\,,$$

$$\frac{1}{4\pi}\sum_{i=1}^{N-1} (\sigma - m_i) \log \frac{|\sigma - m_i|^2 + D}{|\sigma - m_i|^2} + \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} (\sigma - \mu_j) \log \frac{|\sigma - \mu_j|^2 - D}{|\sigma - \mu_j|^2} =$$

$$= (\sigma - m_0)\,|n_0|^2 + (\sigma - \mu_0)\,|\rho_0|^2 + \frac{uN}{4\pi}\,\sigma\,.$$

Solution of (2,2) model

Phase transitions -- artifact of large-N

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0$$

Higgs in n (Hn)

$$\rho_0 = 0 \quad D = -|\sigma - m|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{m}{\Lambda}\,, & \mu < m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m\,. \end{cases}$$

Higgs in rho (Hrho)

$$n_0 = 0 \quad D = |\sigma - \mu|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu < m \end{cases}$$

Coulomb (C)

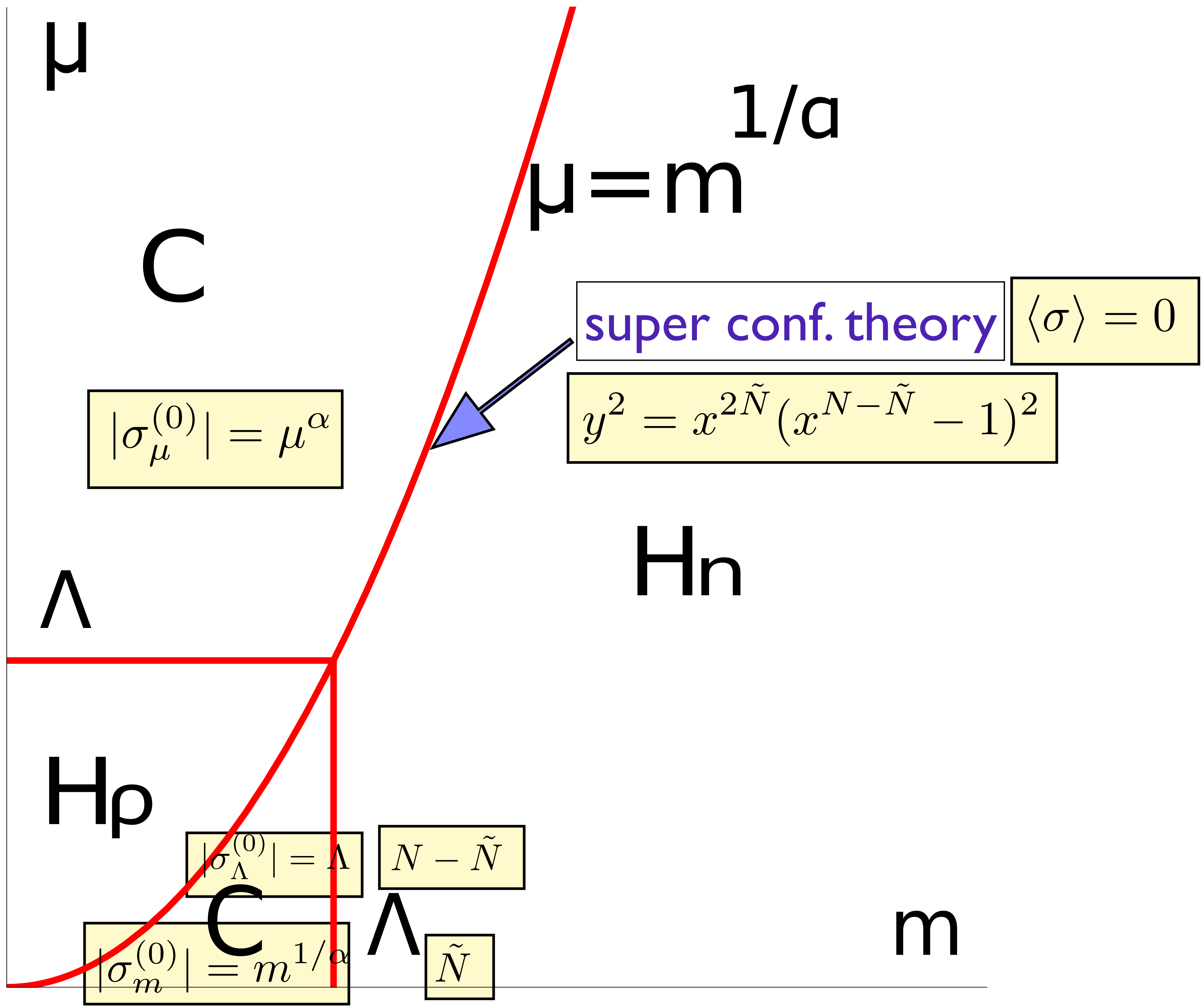
$$n_0 = \rho_0 = 0$$

renormalized FI term vanishes in C phase

in (2,2) from exact superpotential

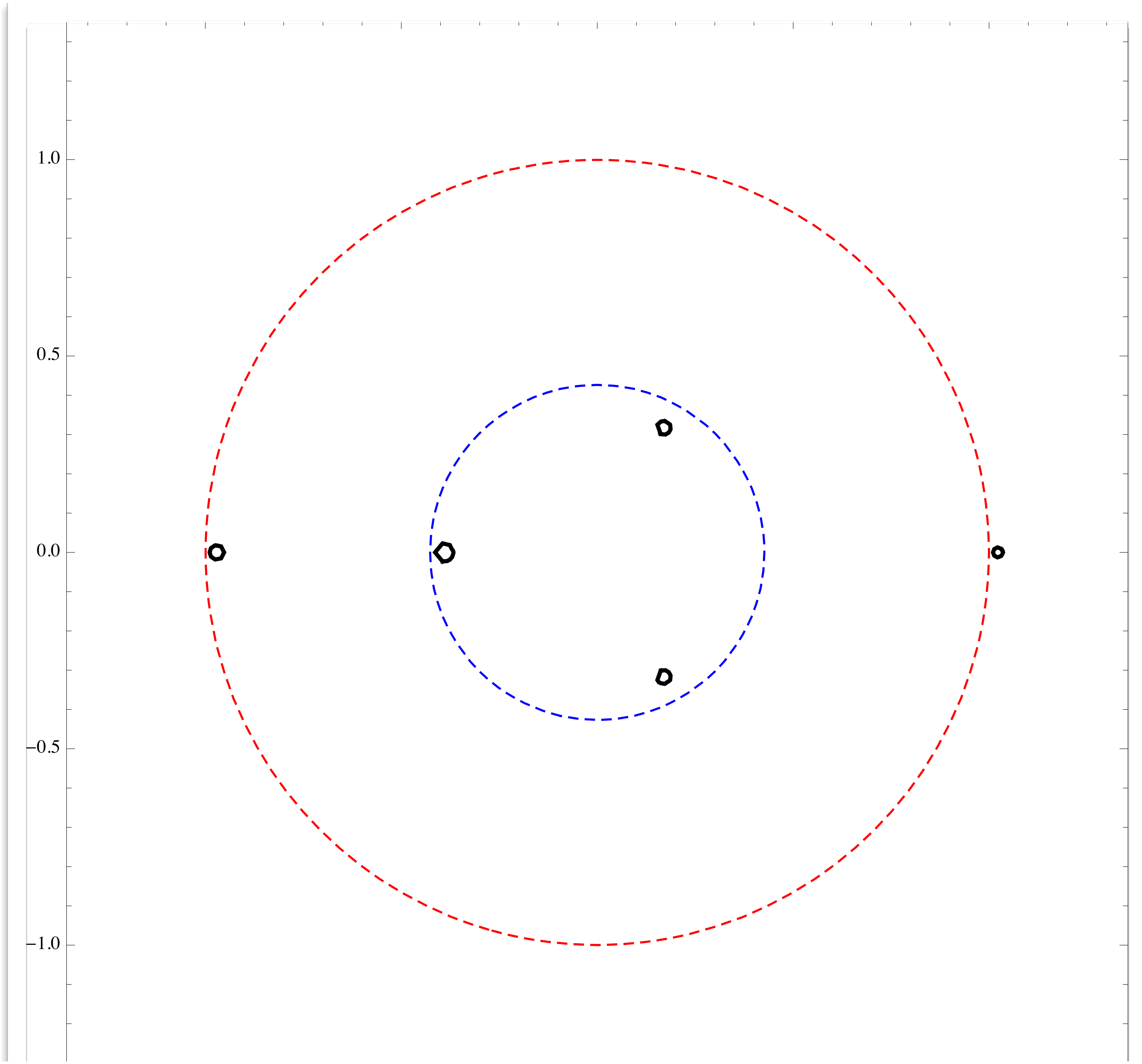
$$\frac{\prod_i (\sigma - m_i)}{\prod_i (\sigma - \mu_j)} = \Lambda^{N-\tilde{N}} \qquad \sigma = 0$$

is one of the solutions...

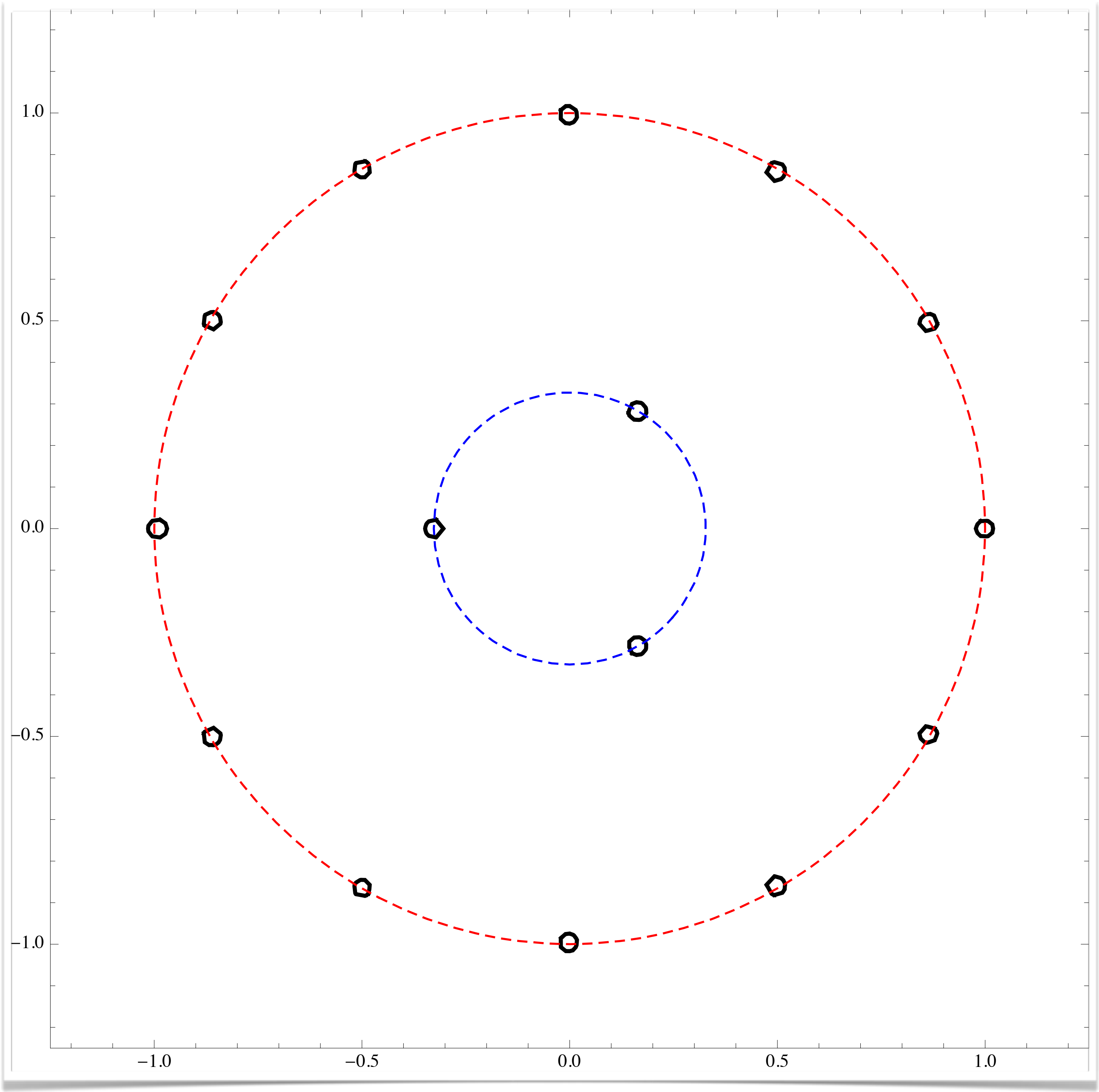


μ
$\mu = m^{1/a}$
C
$|\sigma_\mu^{(0)}| = \mu^\alpha$
super conf. theory
$\langle\sigma\rangle = 0$
$y^2 = x^{2\tilde{N}}(x^{N-\tilde{N}} - 1)^2$
Hn
Λ
Hp
$|\sigma_\Lambda^{(0)}| = \Lambda$
$N - \tilde{N}$
C
$\Lambda_{\tilde{N}}$
$|\sigma_m^{(0)}| = m^{1/\alpha}$
m

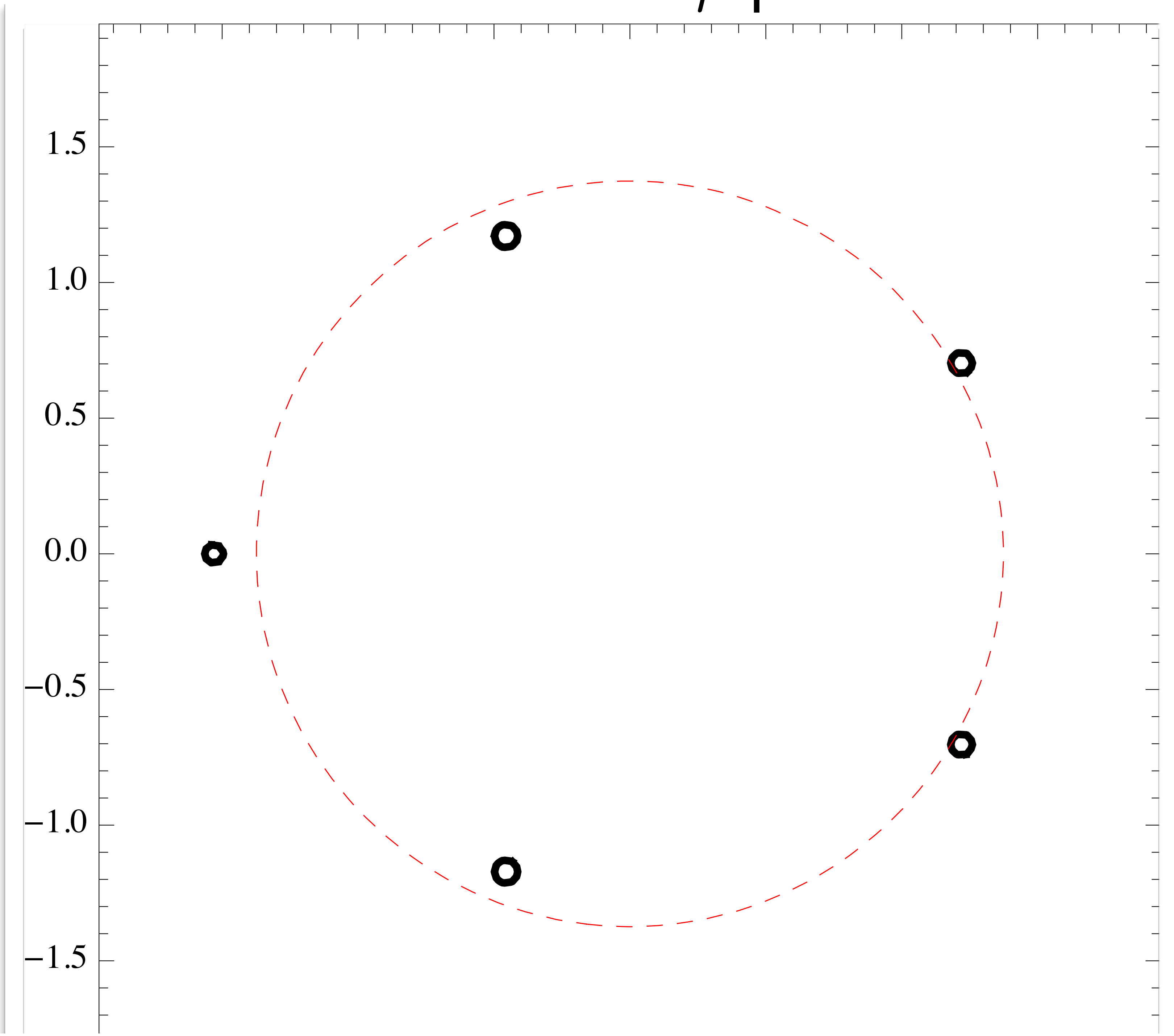
N=5 Nf=8

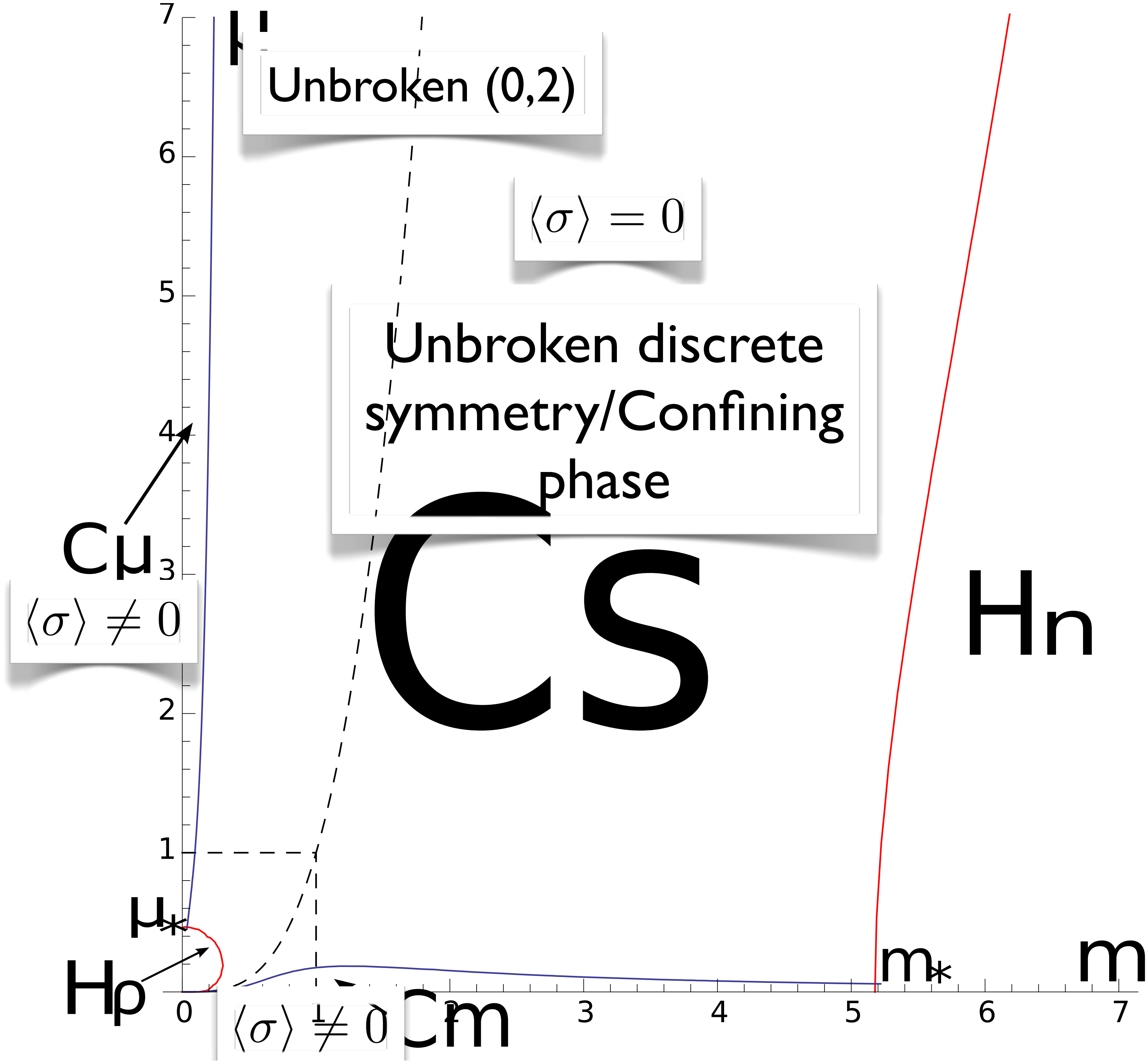


N=15 Nf=18



Nf=5 Cμ phase





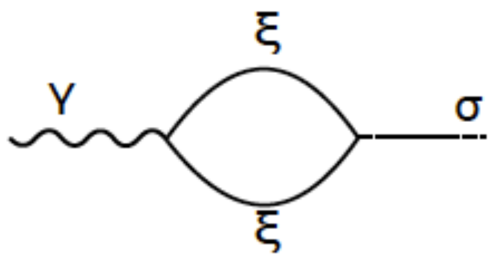
Unbroken (0,2)
$\langle\sigma\rangle = 0$
Unbroken discrete symmetry/Confining phase
Cs
Hn
Cμ
$\langle\sigma\rangle \neq 0$
μ*
Hρ
$\langle\sigma\rangle \neq 0$
Cm
m*
m

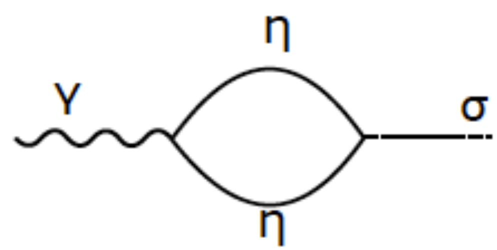
Spectrum

[Bolokhov Shifman Yung]
[PK Monin Vinci]

$$\mathcal{L} = -\frac{1}{4e_\gamma^2}F_{\mu\nu}^2 + \frac{1}{e_{\sigma\,1}^2}(\partial_\mu \mathfrak{Re}\,\sigma)^2 + \frac{1}{e_{\sigma\,2}^2}(\partial_\mu \mathfrak{Im}\,\sigma)^2 + i\mathfrak{Im}(\bar{b}\,\delta\sigma)\epsilon_{\mu\nu}F^{\mu\nu} - V_{\text{eff}}(\sigma) + \text{Fermions}$$

Anomaly





$$b = \frac{N}{4\pi}\left(\frac{1}{N}\sum_{i=1}^{N-1}\frac{1}{\bar{\sigma}_0 - \bar{m}_i} - \alpha\frac{1}{\tilde{N}}\sum_{i=1}^{\tilde{N}-1}\frac{1}{\bar{\sigma}_0 - \bar{\mu}_i}\right)$$

$$m_\gamma = e_{\sigma\,2}e_\gamma|b|$$

Photon becomes massless in Cs phase!! Confinement!

Note that Lambda vacua disappear at large deformations
Need to sit in zero-vacua

e.g. in Cm phase

$$m_\gamma = \sqrt{6}\,\Lambda\left(\frac{\Lambda}{m}\right)^{1/\alpha}\left(\left(\frac{m}{\Lambda}\right)^{2/\alpha} - \left(\frac{\mu}{\Lambda}\right)^2 e^{u/\alpha}\right)\mathrm{e}^{-\frac{u}{2\alpha}}$$

Massless goldstino in fermionic sector

Conclusions and open questions

- Study BPS (and beyond) spectrum of SQCD can effectively be done using 2d NLSM (and GLSM)
- Rich variety of phases in (0,2) model at strong coupling
- Other heterotic deformations $\bar{D}\Phi_+ \sim \bar{D}\Phi_-$
- Are there flux tubes in theories without FI term? (e.g. SU(N)) Omega deformed 4d theory may have such solutions...
- Connections to integrable systems in 2d...
- Relationship w/ another 4d/2d duality [Vafa et al]